SECOND ORDER IDENTIFICATION FILTER

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Abstract. The identification process shows the internal dynamic states based on a reference system commonly known as a black box scheme, built on the transition and gain functions, and an innovation process. Unfortunately, in this sense, the exponential transition function considers the unknown internal parameters. This means that the internal states description does not operate with these conditions considering the internal dynamic gains inaccessible. Against this difficult there is an approximation using the estimation technique. This paper presents estimation for a single input - single output (SISO) model with two delays known as a second order system. This is a special technique which describes the black box internal gains, allowing the transition function to have a sense of identification. The estimator built on the second probability moment form has an adaptive strategy allowing a sufficient convergence rate into identification results. The filter is integrated with two actions, estimation and internal states description, seeking a good convergence level considering the gradient description. Therefore, the theoretical result demonstrates the filter strategies.

Keywords: Digital Filter, Estimation, Functional Error, Identification, Stochastic Gradient, Reference Model.

1. Introduction

A physical system must be performed as a model mathematically validated with different processes. The difference between the real output system and its mathematical output representation tends to be a predefined bounded region. The model as a black box scheme establishes a relationship between the output and input signals system. In particular, the internal states are non-observables and are related mathematically between the input and output signals through the transfer function.

The system viewed as a black box permits the transfer function without access to their internal dynamics in a direct form. In this case the filter process is necessary describing the states indirectly; i.e., the identification is a technique describing the internal states [1-2]. This system commonly gives the output online without knowing what happens inside at a specific time. Therefore, the techniques were developed and applied into the identification filter structure [3]. The internal dynamics description was based on the early work of identification depending on the transition function [4], which in its simplest form obtains the primitive equation including the internal gain [5]. The digital filter identification can be seen illustratively in Figure 1.

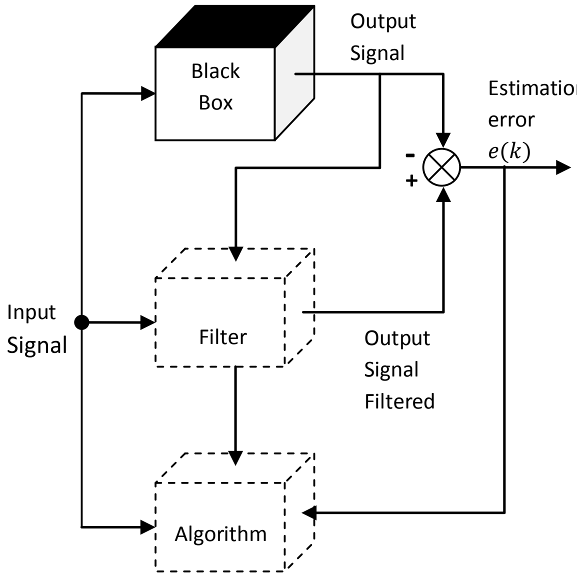

Figure. 1. Black Box and Digital Filter.

The black box system evolution is achieved through interpolation methods approximating the response to a real output system [6]. The parameter function plays an important role in the convergence rate considering it affects the model evolutions [7].

In control theory, equations are commonly used in the states space meaning the mathematical model depends on internal states and parameters [8]. In spite of this, internal unknown conditions are described using combined estimation and identification techniques [9]. The output velocity state changes with bounded movements, so that the model proposed corresponds to a second order with two delay states [10]. Therefore, the parameter estimation in state spaces in discrete form is applied into an identification model [10]. Once estimated and adapted to an identification technique, the evolution adjusts the estimation results converging the output identification filter to the desired output answer [11].

The estimation technique is classified into recursive and non-recursive. A Non-recursive technique corresponds to a final parameter vector. A recursive techniques process evaluates the evolution parameters through time [12]. Once parameters are obtained, the transition function is built [13].

The internal dynamics reference system in simple form considers the input and output bounded signals viewed as linear and stationary in the probability sense [14], with unknown parameters [15, 16]. The traditional identification process is not feasible if the gain used for the transition function is inside the black box [17].

The transition function being applied in the adaptive process is based on: 1) the initial conditions of the transition function. 2) The second probability identification error moment form. 3) A criterion dynamically modifies the gain.

The identification error was obtained by the difference between the reference system response and the filter. To achieve the identification, it is necessary to know the transition role and parameters through the second probability moment. Once internal parameters are estimated, the transition function is built to be applied in identifying the internal states. The identifier signal and the functional error built on the basis of the second probability moment correct the estimation dynamically, making the interaction lead to the filter in the convergence region.

The estimation and identification are related by the functional error, obtaining a dynamic digital filter. According to the following order, the retrieved estimation is a gradient stochastic type accessing the system output. The identification is described by the recursive structure and also, the functional error. The estimation and identification findings are presented. In general, the importance of the transition in identification is solved using the estimation as an adaptive procedure.

2. Theoretical Analysis

The estimation describes the black box internal parameters system, but is it possible to estimate parameters that affect the identification filter? The answer is yes, because the transition function affects the convergence between the identification and reference signal. Therefore, the estimation would be part of the identification considering the transition function built with estimation parameters. The adaptive process is built considering that the estimation is based on functional identification error affecting the gains in direct form as shown in Figure 2 using the symbols of Table 1, described in finite differences.

Table 1. Black box symbols

$\longrightarrow$	Input, From input to output, Output,
z^{-1}	Signal delay , $x_{k-1} := x_k z^{-1}$
$\otimes$ (+, +)	Algebraic adding signals
$\bullet$	Algebraic multiplicity signals

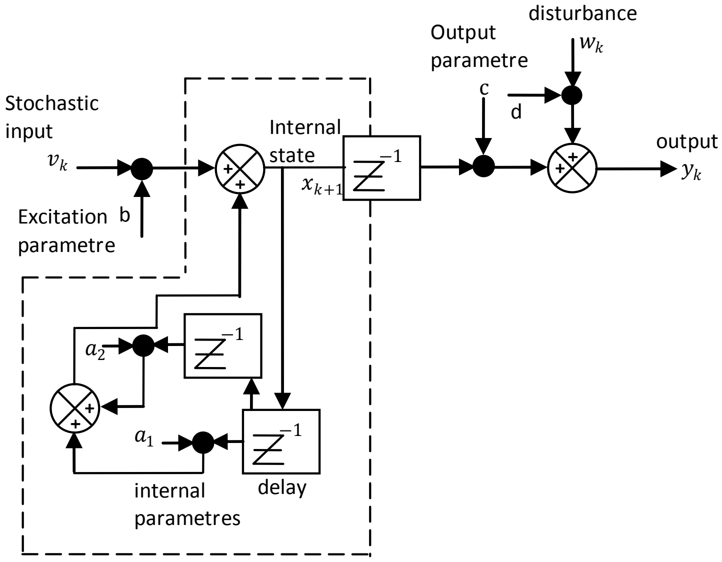

Figure. 2. Filter Estimation Scheme.

Figure 2 considers the output state space (v_k) with external perturbation (w_k). Inside the box with a dashed line is known as dynamic internal parameters described in state space recursive form.

The reference system according to [18] has a recursive form (1) in which the state space according to [19-21] is a function of $x_k = f(a_1, a_2, x_{k-1}, x_{k-2}, b, v_k)$; with output $y_k = f(c, x_k, d, w_k)$, as shown in Figure 2, where $\{w_k := w(k), k \geq 0\}$ is a stochastic description bounded by a Gaussian process $N(\mu = k_{w_k}, \sigma^2_{w_k} < \infty)$. , and $\{v_k := v(k), k \geq 0\}$ with $N(\mu = k_v, \sigma^2_w < \infty)$.

In Figure 2, the identification filter requires knowing the estimated internal parameters. Then the identification filter includes the estimation technique implemented as a system.

Theorem 1 (Adaptive Identification). Consider a black box system described by a second-order stochastic model expressed in finite differences (1)

$y_k = c_k x_k + w_k$ with an internal state $x_k = a_1 x_{k-1} + a_2 x_{k-2} + b v_k$. (1)

Bounded in time evolution $\tau_k < \infty$ according to [22] and $f_{\max_k}$ is the frequency representing either bounded $f_{\max_k} < \infty$ [23] with respect to $\{w(k), k \geq 0\}$ and $\{v(k), k \geq 0\}$ such as discrete stochastic processes described by $N(\mu_w = k_w, \sigma_w^2 < \infty)$, $k_w \in \Re_+$, $N(\mu_v = k_v, \sigma_w^2 < \infty)$, $k_v \in \Re_+$, and satisfying $x_k, x_{k+1}, y_k \in \Re_+^{[n\times 1],k}$; with parameters $a_1,\ a_2 \in \Re_+$ based on an error identification and its variance with respect to [20-23] and $\hat{y}_k$ described in (2).

$$\hat{y}_k = \hat{a}_1 y_{k-1} + \hat{a}_2 y_{k-2} + \tilde{d}\tilde{w}_k \tag{2}$$

with

$$\hat{a}_{1,k} = \frac{E\{\hat{y}_k y_{k-1}\})}{E\{y_{k-1}^2\}},\ \hat{a}_{2,k} = \frac{E\{\hat{y}_k y_{k-2}\})}{E\{y_{k-2}^2\}}$$

Proof (Theorem 1): Considering the recursive functional error in (3).

$$J_k = \tfrac{1}{k}(E\{e_k^2\} + (k-1)J_{k-1}). \tag{3}$$

The identification error described in (4):

$$e_k := y_k - \hat{y}_k \tag{4}$$

According to the system (1), the output signal is described in (5).

$$y_k = c(a_1 x_{k-1} + a_{2,} x_{k-2} + bv_{k-1}) + dw_k \tag{5}$$

With one and two delays, the output signal is (6):

$$y_{k-1} = cx_{k-1} + dw_{k-1}\ ,\ y_{k-2} = cx_{k-2} + dw_{k-2} \tag{6}$$

The unobservable states for y_{k-1} and y_{k-2} are solved in (7).

$$\begin{aligned} x_{k-1} &= c^{-1} y_{k-1} - c^{-1} dw_{k-1} \\ x_{k-2} &= c^{-1} y_{k-2} - c^{-1} dw_{k-2} \end{aligned} \tag{7}$$

Substituting the equation (7) in (5) has (8):

$$y_k = c(a_{1,k-1}(c^{-1} y_{k-1} - c^{-1} dw_{k-1}) + a_{2,k-1}(c^{-1} y_{k-2} - c^{-1} dw_{k-2}) + bv_{k-1}) + dw_k$$

$$y_k = ca_1 c^{-1} y_{k-1} + c_k a_2 c^{-1} y_{k-2} - ca_1 c^{-1} dw_{k-1} - ca_2 c^{-1} dw_{k-2} + cbv_{k-1} + dw_k \tag{8}$$

Reducing terms as shown in (9)

$$y_k = \tilde{a}_1 y_{k-1} + \tilde{a}_2 y_{k-2} + \tilde{d}\tilde{w}_k \tag{9}$$

Where the Gaussian general noise (10)

$$\tilde{d}\tilde{w}_k = -a_1\tilde{d}\tilde{w}_{k-1} - a_2\tilde{d}\tilde{w}_{k-2} + cbv_{k-1} + \tilde{d}\tilde{w}_k,\ \tilde{a}_1 = a_1,\ \tilde{a}_2 = a_2 \tag{10}$$

Considering the equation (9) in (4) the identification error has the form (11):

$$e_k = \tilde{a}_1 y_{k-1} + \tilde{a}_2 y_{k-2} + \tilde{d}\tilde{w}_k - \hat{y}_k \tag{11}$$

Developing (11) in (3) obtains the functional error (12):

$$\begin{gathered} J(e_k^2) = \tilde{a}_1^2 E\{y_{k-1}^2\} + \tilde{a}_2^2 E y_{k-2}^2 + \tilde{d}_k^2 E\{\tilde{w}_k^2\} + E\{\hat{y}_k^2\} - \\ 2[(\tilde{a}_1 E\{\hat{y}_k y_{k-1}\} + \tilde{a}_2 E\{\hat{y}_k y_{k-2}\} + \tilde{d}_k E\{\hat{y}_k \tilde{w}_k\}) + \\ ((\tilde{a}_2\tilde{d})E\{y_{k-2}\tilde{w}_k\} + (\tilde{a}_1\tilde{a}_2)E\{y_{k-1}y_{k-2}\} + (\tilde{a}_1\tilde{d})E\{y_{k-1}\tilde{w}_k\})] \end{gathered} \tag{12}$$

The stochastic gradient of (12) with respect to $\tilde{a}_1$ achieved in (13).

$$\nabla J(e_k^2)_{\tilde{a}_1} = 2\tilde{a}_1 E\{y_{k-1}^2\} - 2[(E\{\hat{y}_k y_{k-1}\}) + (\tilde{a}_2 E\{y_{k-1}y_{k-2}\} + dE\{y_{k-1}\tilde{w}_k\})] \tag{13}$$

From (13) the estimated value $\tilde{a}_1$ considering $E\{y_{k-1}y_{k-2}\} = 0$, $E\{y_{k-1}\tilde{w}_k\} = 0$ is described in (14)

$$\hat{\tilde{a}}_{1,k} = \frac{E\{\hat{y}_k y_{k-1}\})}{E\{y_{k-1}^2\}} \tag{14}$$

The stochastic gradient of (13) with respect to $\hat{a}_2$, is shown in (15)

$$\hat{\tilde{a}}_{2,k} = \frac{E\{\hat{y}_k y_{k-2}\})}{E\{y_{k-2}^2\}} \tag{15}$$

The adaptive identification finally has the form (16)

$$\hat{y}_k = \hat{\tilde{a}}_{1,k} y_{k-1} + \hat{\tilde{a}}_{2,k} y_{k-2} + \tilde{d}\tilde{w}_k . \blacksquare \tag{16}$$

3. Conclusion

In the black box concept, the exponential transition function is inaccessible considering the unknown internal parameters and the identification does not operate with these conditions.

The estimation technique based on stochastic gradient solves this difficulty. The system considered was a single input - single output (SISO) model with two delays, described as a second order structure. The estimation parameters were built on the second probability moment form and have adaptive strategies allowing a sufficient convergence rate in the gradient sense. Finally, the identification was integrated with the both estimations into a traditional identification. The theoretical result described the filter strategies integrated as an adaptive filter structure.

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